Econ 802

First Midterm Exam

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All questions have equal weight. It is a good idea to read the entire exam before you start writing. You may want to work first on the questions where you feel most confident.

1. Acme Inc. has the production function $y = \max\{0, a + bx\}$ where $y \geq 0$ and $x \geq 0$ are scalars and $b > 0$. The output price is $p > 0$ and the input price is $w > 0$.

(a) Suppose $a < 0$. Determine whether Acme has decreasing, constant, or increasing returns to scale and justify your answer. Then show the production function on a graph with x on the horizontal axis and y on the vertical axis. Use isoprofit lines involving the prices (p, w) to describe the price vectors (if any) for which there is a solution to the profit maximization problem. If a solution does exist, indicate it on the graph, say whether it is unique, and explain your reasoning.

(b) Repeat your analysis from part (a) for the case where $a = 0$.

(c) Repeat your analysis from part (a) for the case where $a > 0$.

2. Two Stage Enterprises uses the inputs $(x_1, x_2) \geq 0$ to obtain an intermediate input z according to the production function $z = ax_1 + bx_2$ where $a > 0$ and $b > 0$. Then it uses z to obtain the final output $y \geq 0$ according to the production function $y = z^4$.

(a) Assume free disposal of all inputs and outputs. Consider production plans of the form $(y, -x_1, -x_2)$. Is the production possibilities set Y convex? strictly convex? non-convex? Justify your answers using some algebra and a graph.

(b) Let $p > 0$ be the price of y and $w = (w_1, w_2) > 0$ be the prices of (x_1, x_2). Ignoring the non-negativity constraints (don't use Kuhn-Tucker multipliers), derive the first order conditions for the problem $\max\{p(ax_1 + bx_2)^4 - w_1x_1 - w_2x_2\}$. Would it ever be possible to satisfy these conditions? Justify and interpret your answer.

(c) Derive the necessary second order condition for the same problem. Would it ever be possible to satisfy this condition? Justify and interpret your answer.

3. Define the technical rate of substitution (TRS) at a point $x = (x_1, x_2) \geq 0$ to be the slope of the isoquant passing through that point.

(a) We usually write conditional input demand functions in the form $x_1(w_1, w_2, y)$ and $x_2(w_1, w_2, y)$. TRS is not an argument in these functions. However, people often talk about the derivative $\partial(x_1/x_2) / \partial|\text{TRS}|$. Under what conditions, if any, does this derivative made sense? Explain carefully.

(b) Consider the CES production function $y = (x_1^a + x_2^a)^{1/a}$. Describe the relationship between the parameter a and the technical rate of substitution. Explain carefully.

(c) Consider the CES production function and a Cobb-Douglas production function. Use the first order conditions for cost min to show that under certain conditions, the two functions lead to an identical relationship between x_1/x_2 and w_1/w_2 (you can ignore the SOC). Briefly interpret this result in words.

4. Linear Associates can use either of two production functions: $y = Ax_1 + Bx_2$ or $y = Cx_1 + Dx_2$ where all inputs and outputs are non-negative and all capital letters are positive. However, it has to choose just one of the two production functions (it cannot use both simultaneously). Assume free disposal of output.

(a) For a fixed output $y > 0$, assume the isoquants from the two production functions intersect at an interior point $x^0 > 0$ and have different slopes. Draw a graph of the input requirement set V(y). Is the set V(y) monotonic? convex? closed? Explain briefly in each case.

(b) Let the input prices be $(w_1, w_2) > 0$. The firm minimizes the cost of producing the fixed output $y > 0$ from part (a). Draw a graph showing how the firm's behavior changes depending on the prices. Then give a full description of the conditional input demands $x_1(w, y)$ and $x_2(w.y)$.

(c) Suppose Professor Z does not know the true V(y) set but does observe the firm's input choices in response to many different price vectors. What would Professor Z probably conclude about the inner bound VI(y)? What would Prof Z probably conclude about the outer bound VO(y)? Explain using a graph.

5. Here are some miscellaneous questions.

(a) A firm with many inputs and many outputs faces the price vector $p^* > 0$ and uses the production plan y^*. When all of the prices are multiplied by the same scalar $t > 0$, the firm switches to a new production plan $y' \neq y^*$. Could this be consistent with profit maximization? Explain your reasoning.

(b) Consider the generalized Leontief production function $y = \min \{a_1x_1 \,.\,.\, a_nx_n\}$ where $(x_1 \,.\,.\, x_n) \geq 0$ and $a_i > 0$ for all $i = 1 \,.\,.\, n$. The input prices are $(w_1 \,.\,.\, w_n) > 0$. Find the conditional input demands $x_i(w, y)$ and the cost function $c(w, y)$. Explain your reasoning.

(c) There are three ways to get comparative static results for the unconditional input demands $x(p, w)$ where $p > 0$ is the output price and $w > 0$ are the input prices: (i) using the first order conditions, (ii) using the algebraic approach, and (iii) using Hotelling's Lemma. Discuss the advantages and disadvantages of each method.